



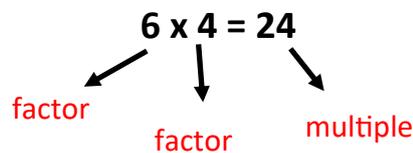
Multiplication and division

Children are expected to:

- ◆ Identify multiples and factors, including finding all the factor pairs of a number and common factors of two numbers.

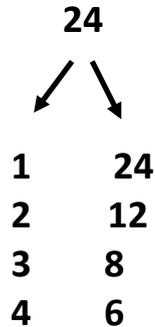
Children should be able to use the vocabulary 'multiple' and 'factor' correctly and consistently.

Factors are numbers that can be multiplied together to make a multiple e.g.



6 and 4 are factors of 24. 24 is a multiple of 6 and a multiple of 4.

A number can have several factors:



So the factors of 24 are: 1, 2, 3, 4, 6, 8, 12 and 24.

Some numbers are factors of more than one number for example:



1 and 7 are common factors of 21 and 14.

Children are taught to find the factors of a number by listing the factor pairs in this way.

They know that they have found them all when a pair is written in reverse e.g. the next pair in the list for 24 would be 6 and 4 but this is already written in reverse.

Children also know to miss out a number if it does not multiply with another number to create the multiple e.g. 24 is not in the 5 times table therefore 5 is not a factor of 24.



Multiplication and division

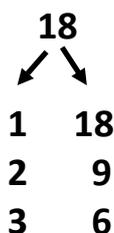
- ◆ Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- ◆ Establish whether a number up to 100 is prime and recall prime numbers up to 19.

A **prime number** is a number that **only has 2 factors: 1 and itself**. In other words, a prime number is a number that can only be divided evenly by 1 and itself.

Any number that is **not a prime number** is a **composite number** as it has more than 2 factors and can be divided evenly by numbers other than 1 and itself.

The **number 1** is the only exception to this: it is considered **neither a prime or composite number** as it only has 1 factor.

A **prime factor** is a number that is a factor of a number and is also prime.



2 and 3 are factors of 18 and are also prime numbers therefore 2 and 3 are prime factors.

Prime numbers are often considered the 'basic building blocks' of the number system since all composite numbers can be made up of prime numbers multiplied together e.g. $12 = 2 \times 2 \times 3$.

Exploring prime numbers can form the basis of mathematical investigations for children. They are expected to know the first prime numbers up to 19 as a known fact.

Prime numbers to 19: 2, 3, 5, 7, 11, 13, 17 and 19.

They are also expected to use a method to establish whether any number up to 100 is prime.

This could be writing out the factors of a number or, an alternative and more efficient method is to use known facts and generalisations to establish whether a number is not prime e.g.

- any even number except the number 2 cannot be prime as they can all be divided by 2
- Any number except 5 that ends with a 5 or a 0 cannot be prime as they can all be divided by 5.
- 93 cannot be prime because 3×10 is 30 so 3×30 is 90 and 1×3 is 3 so 93 can be divided by 3



Year 5



Multiplication and division

- ◆ Multiply numbers up to four-digits by a one or two digit number using a formal written method, including long multiplication for two-digit numbers.

Progressing from the method learnt in Year 4, children are expected to apply the same method to calculations involving multiplying a four-digit number by a one-digit number (see Year 4 policy for a detailed explanation of the short multiplication method).

Initially, children may want to revert back to the expanded method until they feel more confident to solve problems using the short multiplication method.

expanded method

$$\begin{array}{r}
 3624 \\
 \times \quad 4 \\
 \hline
 14496
 \end{array}$$

(4×4)
 (4×20)
 (4×600)
 (4×3000)

short multiplication method

$$\begin{array}{r}
 3624 \\
 \times \quad 4 \\
 \hline
 14496
 \end{array}$$

Following this, children are taught to multiply two, three and four digit numbers by a two-digit number using a method known as **long multiplication**.

The long multiplication method, follows the same process as the short multiplication method however there is an additional step to complete.

Initially, children may prefer to do this using an expanded method however this quickly becomes very time consuming and inefficient and can be more confusing.

Expanded method

$$\begin{array}{r}
 76 \\
 \times 58 \\
 \hline
 48 \quad (8 \times 6) \\
 560 \quad (8 \times 70) \\
 300 \quad (50 \times 6) \\
 3500 \quad (50 \times 70) \\
 \hline
 4408
 \end{array}$$

Short multiplication method

$$\begin{array}{r}
 76 \\
 \times 58 \\
 \hline
 608 \\
 3800 \\
 \hline
 4408
 \end{array}$$



Multiplication and division

Example and explanation of two-digit x two-digit short multiplication.

Key vocabulary:

$453 \times 6 = 2718$
multiplicand multiplier product

Children are initially taught to leave a gap between the answer rows to allow space for the exchanges to be written clearly and avoid unnecessary mistakes (see image). Children are also taught to cross out the exchanges once they have added them on in this method to ensure that they add these at the correct time within the method and do not make unnecessary mistakes. These are strategies that children may choose to move away from once they are secure with the method.

```
  76
x 58
-----
 608
  #
3800
 3
-----
4408
 1
```

1. Multiply the whole multiplicand (the number on the top row) by the ones digit of the multiplier exactly as in the short multiplication method (see Year 4 for an explanation of how to do this if needed).

```
  76
x 58
-----
 608
  #
```

2. Move on to multiplying by the tens digit of the multiplier (in this case the 5 being 50).

To make this easier, put a 0 in the ones column of the answer. This immediately makes the answer ten times bigger and the tens digit can then be treated as one digit. The children are taught to “drop a zero”.

```
  76
x 58
-----
 608
  #
3800
  0
```

3. Now continue the calculation as in short multiplication, working from right to left of the place value columns when placing the digits in the answer. In this case the next step would be 5×6 (this is really 50×6 but because we have dropped a zero we can treat it as 5×6 as the answer has already been made ten times bigger). $5 \times 6 = 30$.

Add on any exchanges and record the answer in the tens column, exchanging into the hundreds if needed.

Cross out the exchanges to make it clear that this has been added on.

In this case, there are no exchanges to add on so the 0 would be recorded in the tens column and the 3 exchanged into the hundreds.

```
  76
x 58
-----
 608
  #
3800
  3
-----
4408
```



Multiplication and division

4. Multiply the next digit in the multiplicand by the multiplier.

In this case this would be 5×7 (this is really 50×70 but can be treated as 5×7 due to the dropped zero). $5 \times 7 = 35$.

Add on any exchanges and record the answer in the hundreds column and thousands column if the answer is a 2 digit number (no need to exchange as this is the final part of the calculation to be multiplied).

Cross out any exchanges to make it clear that these have been added on.

In this case, $35 + 3 = 38$ so the 8 would be recorded in the hundreds column and the 3 in the thousands column.

$$\begin{array}{r} 76 \\ \times 58 \\ \hline 608 \\ \\ \hline 3800 \end{array}$$

5. Draw an equals sign and use column addition to add the answers from the multiplications.

$$\begin{array}{r} 76 \\ \times 58 \\ \hline 608 \\ \\ \hline 3800 \\ \\ \hline 4408 \\ \hline \end{array}$$

Common errors include:

- Writing the digits in the wrong place value column
- Forgetting to exchange
- Forgetting to add on any digits that have been exchanged
- Not leaving appropriate space and becoming muddled within the calculation
- Not crossing out the exchanges and adding these in the final addition part of the calculation
- Not recognising that a calculation is the final one for that row and becoming confused with where to place the digit
- Multiplying the exchange as part of the calculation (this is not necessary as it has already been multiplied and therefore just needs adding on to the answer for the step)
- Calculation errors within the method

These errors are addressed and explicitly pointed out to the children through teacher modelling. The children engage in discussions about these errors and, in doing so, develop a deep understanding of how to complete the method accurately in order to arrive at the correct answer and strategies to be able to identify where they may have made a mistake.



Multiplication and division

Here are examples of long multiplication for multiplying a 3 and 4 digit number by a 2 digit number.

Two examples of long multiplication on grid paper:

$$\begin{array}{r} 324 \\ \times 36 \\ \hline 1944 \\ \times 2 \\ \hline 11664 \end{array}$$
$$\begin{array}{r} 7305 \\ \times 28 \\ \hline 58440 \\ \times 2 \\ \hline 204540 \end{array}$$

- ◆ **Divide numbers with up to four-digits by a one-digit number using a formal written method of short division and interpret remainders appropriately for the context.**

Children should consolidate their understanding of the short division method (bus stop method) and extend their understanding to dividing four-digit numbers. See Year 4 for a detailed explanation of the short division method.

Examples:

Two examples of short division on grid paper:

$$\begin{array}{r} 134 \text{ r } 5 \\ 7 \overline{) 9243} \end{array}$$
$$\begin{array}{r} 0781 \text{ r } 1 \\ 8 \overline{) 6249} \end{array}$$



Year 5



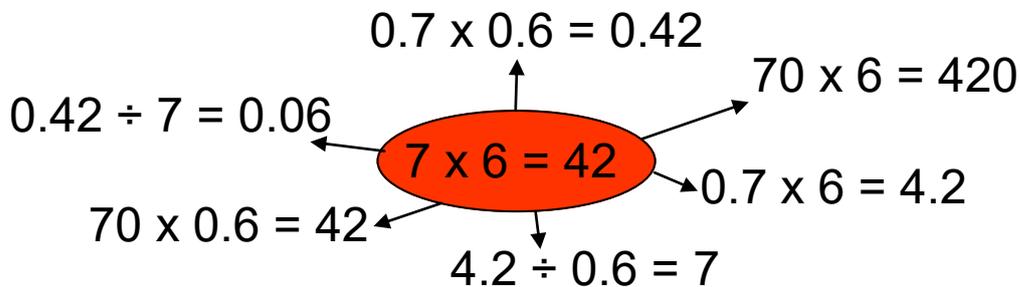
Multiplication and division

- ◆ **Multiply and divide numbers mentally drawing upon known facts.**
- ◆ **Recognise and use square and cube numbers and the notation for squared and cubed.**

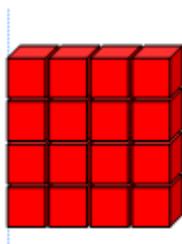
A number of mental strategies should be made available to the children and they should be encouraged to use these in order to be efficient mathematicians. Children should not rely solely on written methods for calculations and should learn to identify when a calculation can be solved easily using a mental method. Daily arithmetic practice in our 'Masters of Arithmetic' sessions encourages the children to be fluent and efficient mathematicians and to identify whether a calculation can be solved using a mental, written or jottings method.

Strategies include:

- Partition simple calculations and complete them mentally
e.g. $35 \times 6 = 30 \times 6 + 5 \times 6 = 180 + 30 = 210$
- Knowledge of certain times table rules e.g. to multiply by 4: double then double again. To multiply by 5: multiply by 10 then halve. To multiply by 20: multiply by 10 then double.
- Knowledge of what happens when we multiply or divide any number by 10, 100 or 1000.
- Knowledge of what happens when we multiply or divide any number by 0.
- Derive related facts from other known facts—see example below:



Children should also recognise square and cube numbers and use the appropriate notation.



$$4^2$$

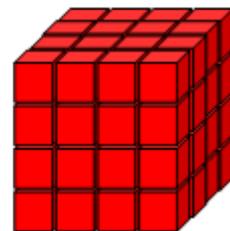
$$= 4 \times 4$$

$$= 16$$

$$4^3$$

$$= 4 \times 4 \times 4$$

$$= 64$$



It is useful for the children to know the first few square and cube numbers in the sequence as known facts:

Squared numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000



Year 5



Multiplication and division

- ◆ **Multiply and divide whole numbers (integers) and those involving decimals, by 10, 100 and 1000.**

It is essential that children are able to mentally multiply and divide by 10, 100 and 1000 and understand what happens to a number when it is multiplied or divided by 10, 100 or 1000.

Initially, children use place value charts and counters or number cards to practically solve problems involving multiplying and dividing by 10, 100 and 1000 until they are confident to solve problems without them.

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
				•			

Children place the number into the chart using counters or number cards. They then move the counters or cards and fill in any empty whole number place value columns with zero as a place holder.

Multiplying—the number gets bigger so the digits move to the left.

Dividing—the number gets smaller so that digits move to the right.

By 10—digits move 1 space

By 100—digits move 2 spaces

By 1000—digits move 3 spaces

(this can be remembered by the number of zeros in 10, 100 and 1000)

Example: 35×10

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
		3	5	.			

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
	3	5	0	.			

Example: $645 \div 100$

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
	6	4	5	.			

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
			6	.	4	5	



Year 5



Multiplication and division

It can be tempting to teach the children to simply add a zero on the end when multiplying by 10 or 2 zeroes when multiplying by 100 however we DO NOT teach the children this as it is incorrect and falls apart when multiplying a decimal number.

Example: 3.8×10

Move the digits.....

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
		3	8	.			

add a zero....

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
			3	.	8	0	

Adding a zero gives the answer of 3.80 which is the same as 3.8 therefore the number has not been multiplied.

Once children have a secure understanding of multiplying and dividing whole numbers and numbers involving decimals by 10, 100 and 1000 they should be able to add this skill to their mental strategies toolkit.

- ◆ Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
- ◆ Solve problems involving a combination of all 4 operations, including understanding the meaning of the equals sign.
- ◆ Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

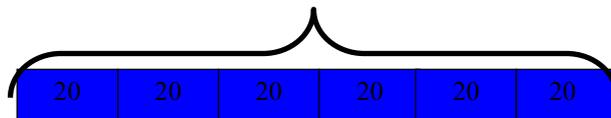
Circle the two multiples of 9 in the list below.

1 3 19 36 90

?

Mr Kirk has 6 boxes of 20 oranges.

How many oranges does he have in total?



$20 \times 6 = 120$ oranges



Year 5



Multiplication and division

Mr Kirk sells half of the oranges.

$$120 \div 2 = 60$$
$$60 \div 5 = 12 \text{ bags}$$

The remaining oranges are packed into bags of 5

How many bags of 5 oranges does Mr Kirk pack?

Scaling up problems such as:

Tom has 67 football stickers.

Tom **67**

Max has 3 times as many stickers as Tom.

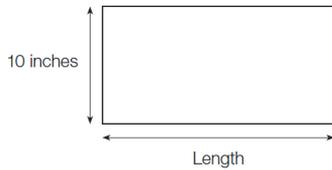
Max **67** **67** **67**

How many stickers do they have altogether?

$$67 \times 4 = 268 \text{ stickers}$$

Children would use short multiplication to find the answer to this problem. They would need to recognise that the question is asking how many they have altogether therefore the calculation is 67×4 not 3. This is a common mistake in questions of this kind.

The length of a paving slab is **double** the width.



$$\text{Length} = \text{double } 10 = 20 \text{ inches.}$$

$$\text{Width} = 2.5 \times 10 = 25 \text{ cm}$$

$$\text{Length} = 2.5 \times 20 = 50 \text{ cm}$$

One inch is approximately 2.5 cm.

Work out the approximate length of the paving slab in centimetres.

Kate's book has 280 pages.



$$1/4 \text{ of } 280 = 280 \div 4 = 70$$

70 pages read on Monday

$$280 - 70 = 210$$

50% of 210 = 105 pages read on Tuesday.

She reads a quarter of the book on Monday.

She reads 50% of the remaining pages on Tuesday.

Calculate how many pages of the book Kate read on Tuesday.

Some children may spot that if she read $1/4$ on Monday then there are $3/4$ left on Tuesday and work out $3/4$ of 280 as 210.

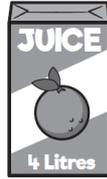


Year 5



Multiplication and division

Eva buys 4 litres of juice.



She drinks 750 ml of the juice herself and shares the rest between her five friends.

How much juice does each friend get?

$$4 \text{ litres} = 4000\text{ml}$$

$$4000 - 750 = 3250\text{ml}$$

$$3250\text{ml} \div 5 = 650\text{ml}$$

Children would likely use short division to solve the final step of this problem.

Children must also know how many ml are in 1l to be able to solve this accurately.

Children also solve problems where the equals sign is acting as a balance e.g.

$$4 \times 6 = 3 \times ?$$

Key vocabulary

See Year 4

Cube number, square number, prime number, factor, factor pair, multiple, prime factor, common factor, composite number, integer, inverse, power of